Efficiency and Risk Trade-offs in Dynamic Oligopoly Markets

Qingqing Huang, Mardavij Roozbehani, Munther Dahleh

Laboratory for Information and Decision Systems

Massachusetts Institute of Technology

[1] Q. Huang, M. Roozbehani, M. Dahleh, "Efficiency and Risk Impacts of Market Architecture in Power Systems under Real-Time Pricing", To Appear in IEEE CDC, 2012
[2] ____, "Efficiency and Risk Trade-offs in Dynamic Oligopoly Markets", work in progress

Motivation

Exogenous shocks VS endogenous risk
Tail risk, aggregate demand / price spikes

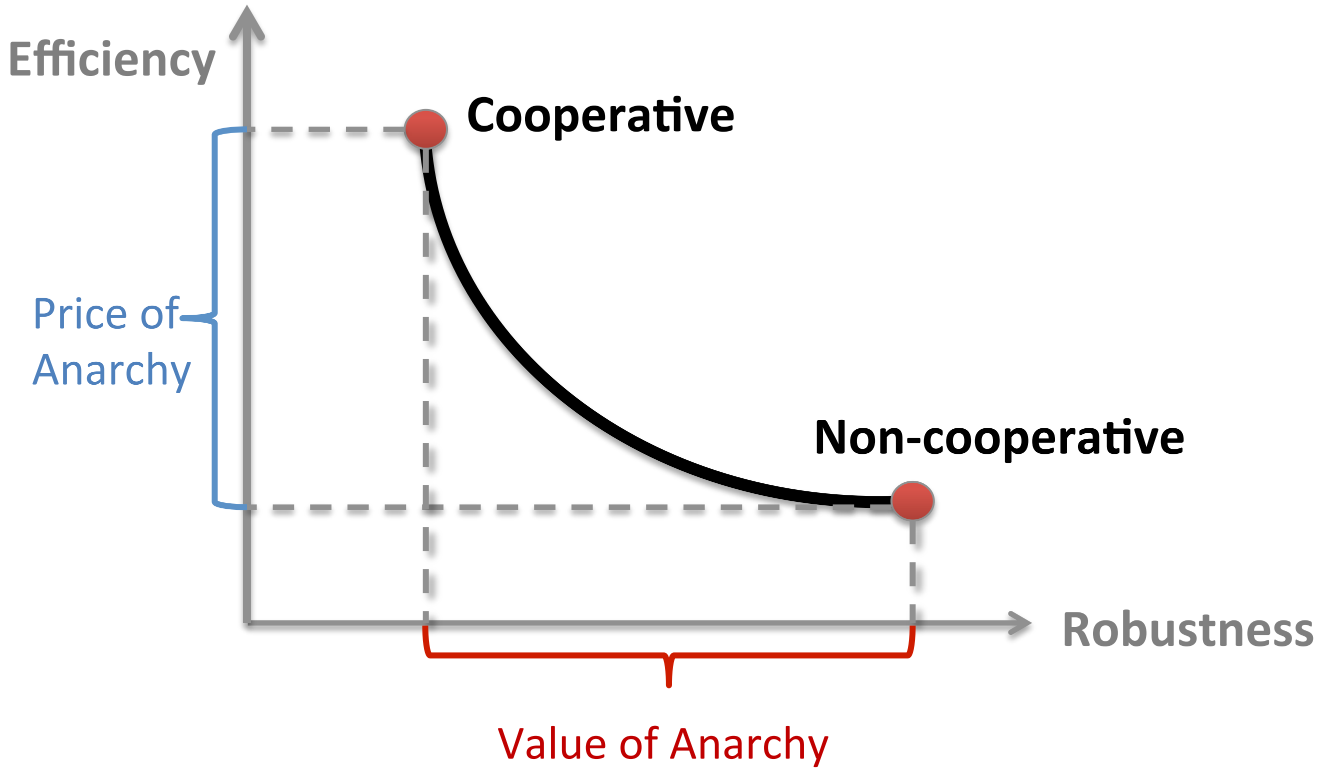
- How does **endogenous risk** emerge in multi-agent dynamic system?
- What's the impact of local incentives on system-wide efficiency and risk?

market architectures → agent behaviors → aggregate outcome

Efficiency VS Risk

Tradeoff

Contribution



cooperation

Reduce externality, more efficient
More vulnerable to some exogenous shocks

system dynamics, uncertainties, frictions (deadline constraints)

Literature

- Previous works on **endogenous risk**
  - Heterogeneous beliefs [1]
  - Failure of the agents to rationalize feedback links [2]

- Our work
  - Agents are fully aware of pricing mechanism,
    have perfect information of system state,
    form rational expectations about other agents in the market.

  - Market architecture → Endogenous risk
  - Tradeoff

- Approach
  - Case study : Dynamic stochastic game, MDP
  - LTI reformulation and tradeoff analysis

[1] The leverage cycle (Geanakoplos 2009)
[2] “Endogenous Risk” (Danielsson and Shin 2003)

Setup

Agent arrival

- **L** types (deadline constraint)
- Uncertainty
  - Bernoulli arrival
  - Workload distribution

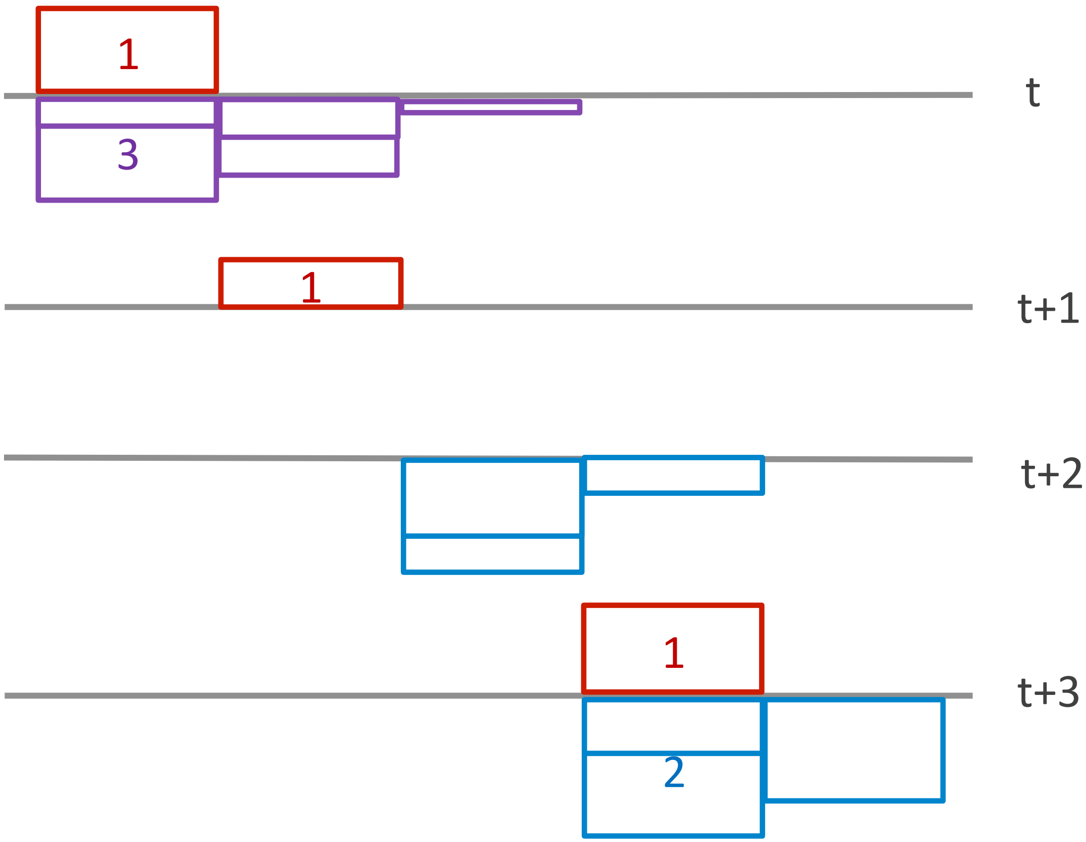
$$d_l(t), l \in \{1, \ldots, L\}$$

Decision

- State information
- Load scheduling

Cost $\mathbf{E}[\sum_{\tau=t}^{t+l} u_i(t)p(t)]$

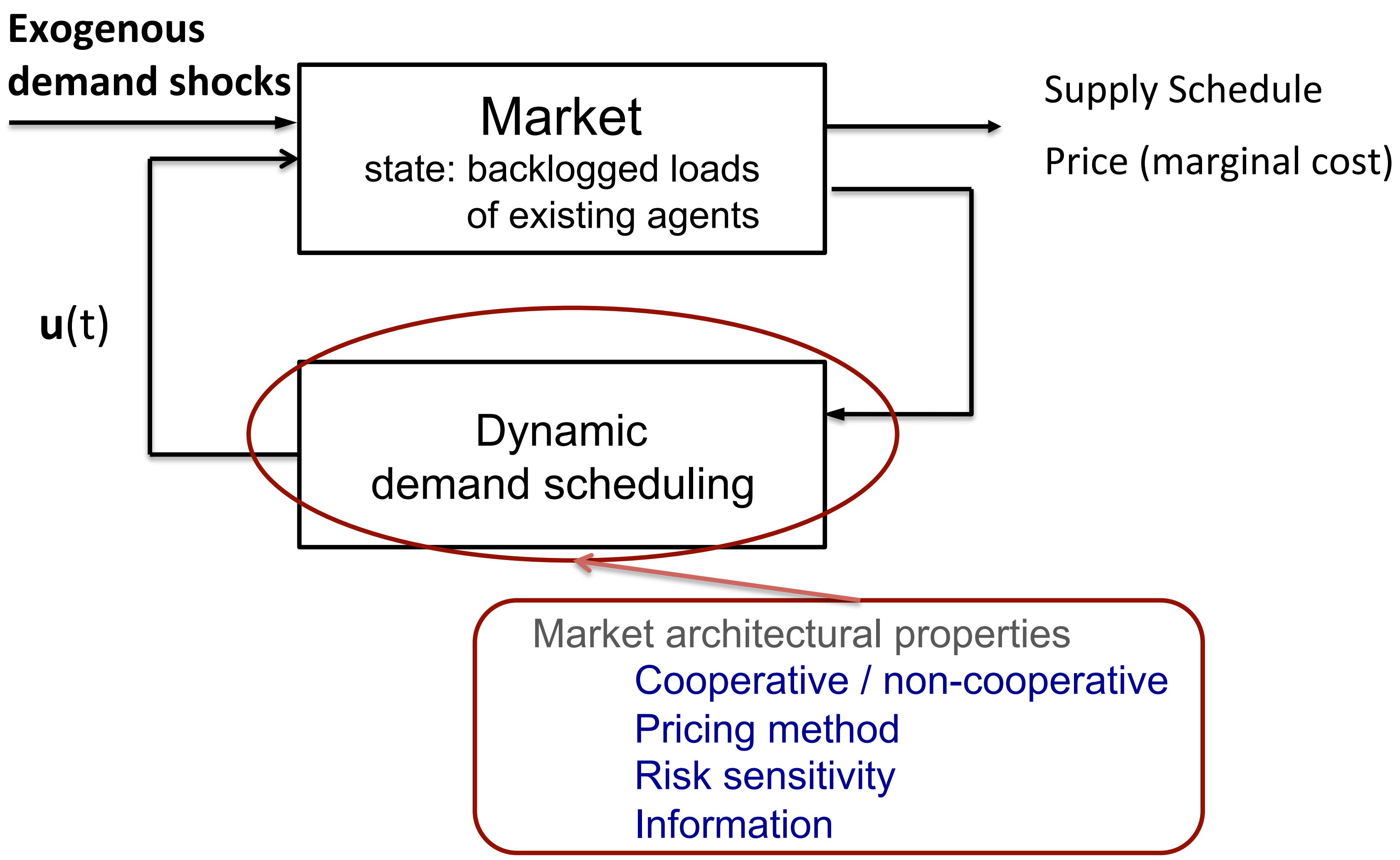
Pricing $p(t) = \sum_i u_i(t)$



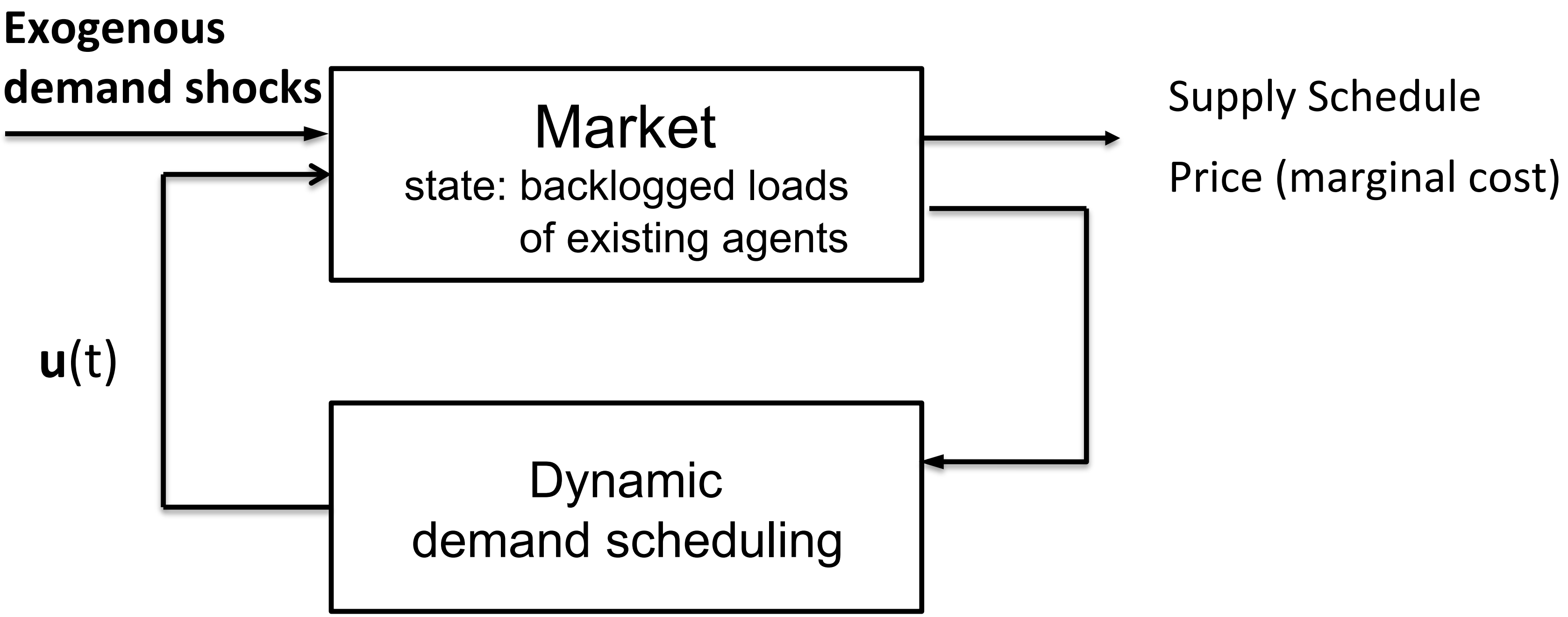
System performance

- **Efficiency: expected average cost** (**variance** of aggregate demand process)
- **Risk: aggregate demand spikes** (**tail probability** of aggregate demand process)

Setup


Exogenous
demand shocks
Market
state: backlogged loads
of existing agents
Supply Schedule
Price (marginal cost)
u(t)
Dynamic
demand scheduling
Market architectural properties
Cooperative / non-cooperative
Pricing method
Risk sensitivity
Information

Setup



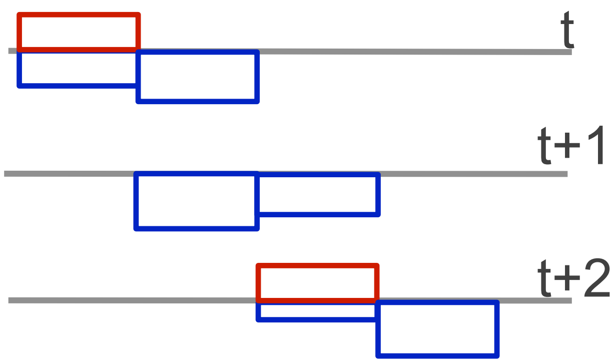
Applications:

- consumer response to real-time pricing in power grids
- load scheduling in cloud computing market
- multi-portfolio execution problem
- consumption risk sharing

Case study L=2

Solutions

market architectures → agent behaviors → aggregate demand process



- At most 1 decision maker at each period

$$\left(u(t),\quad d_2(t) - u(t)\right)$$

- Linear quadratic

$$u^s(x, d_2) = -a^s x + b^s d_2 + e^s$$

Non-cooperative	**Cooperative**
Dynamic stochastic game Markov perfect equilibrium	Infinite horizon average cost MDP

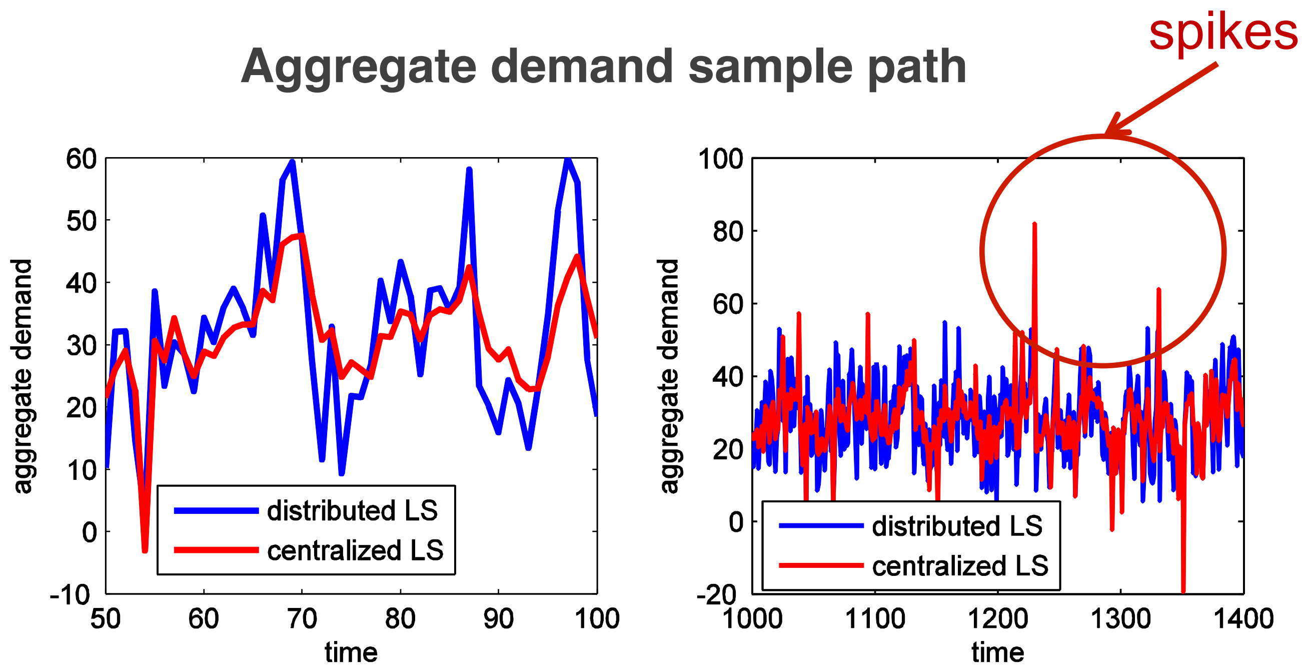
$$a^{nc} < a^c \quad b^{nc} > b^c$$

In cooperative scheme, agents respond more aggressively to other agents' shock

Low efficiency Low risk	High efficiency High risk

Case study L=2

Results



small time scale

large time scale

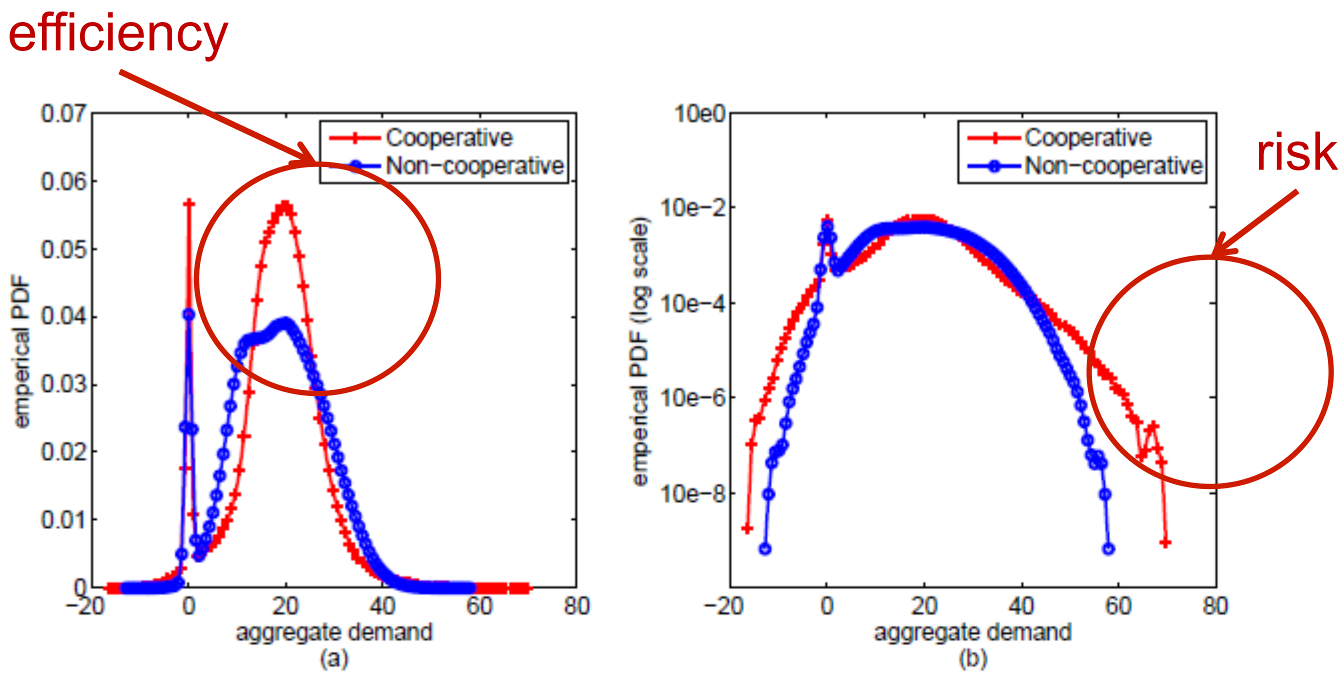
Cooperative

Non-cooperative

Case study L=2

Results

Aggregate demand distribution



Linear scale

Log scale

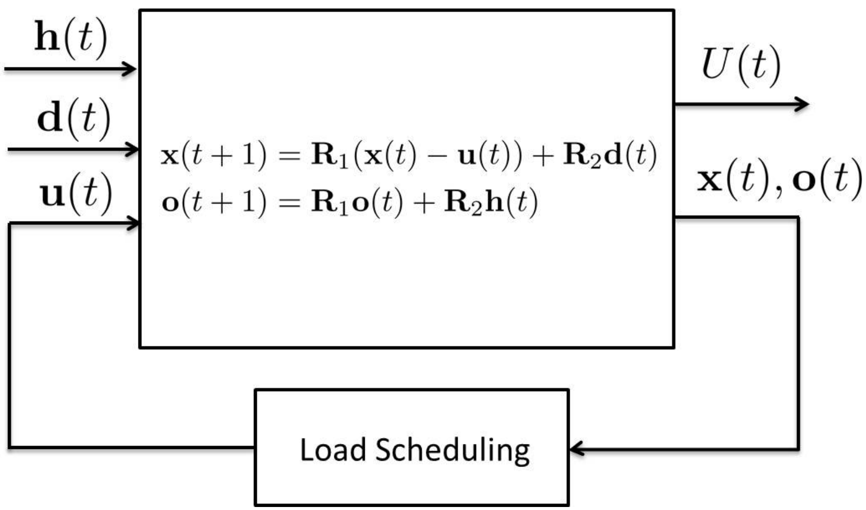
Cooperative

Non-cooperative

General L analysis

Modified system dynamics

- Jump linear system
- No closed form solution



(a)

Risk of spikes

High backlog of load

Absence of flexible load (Bernoulli)

↑

Cooperative load scheduling

less externality

more rely on each other

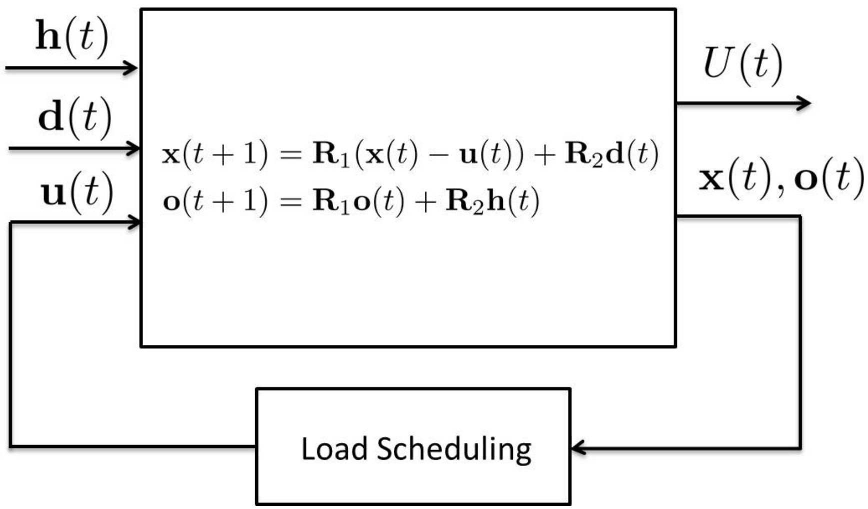
more heavily use of backlog

General L analysis

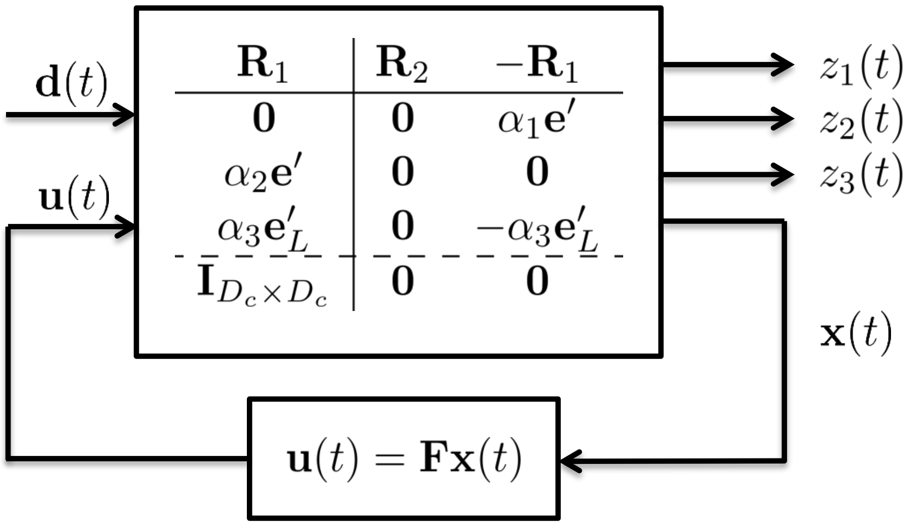
Modified system dynamics

- Focus on linear dynamics
- Relax deadline constraints
- Substitute risk measure

- LTI system
- Multi-objective optimization
  - **Efficiency** aggregate demand 2nd moment
  - **Risk** aggregate backlog 2nd moment
  - **Aggregate unsatisfied load** 2nd moment



(a)

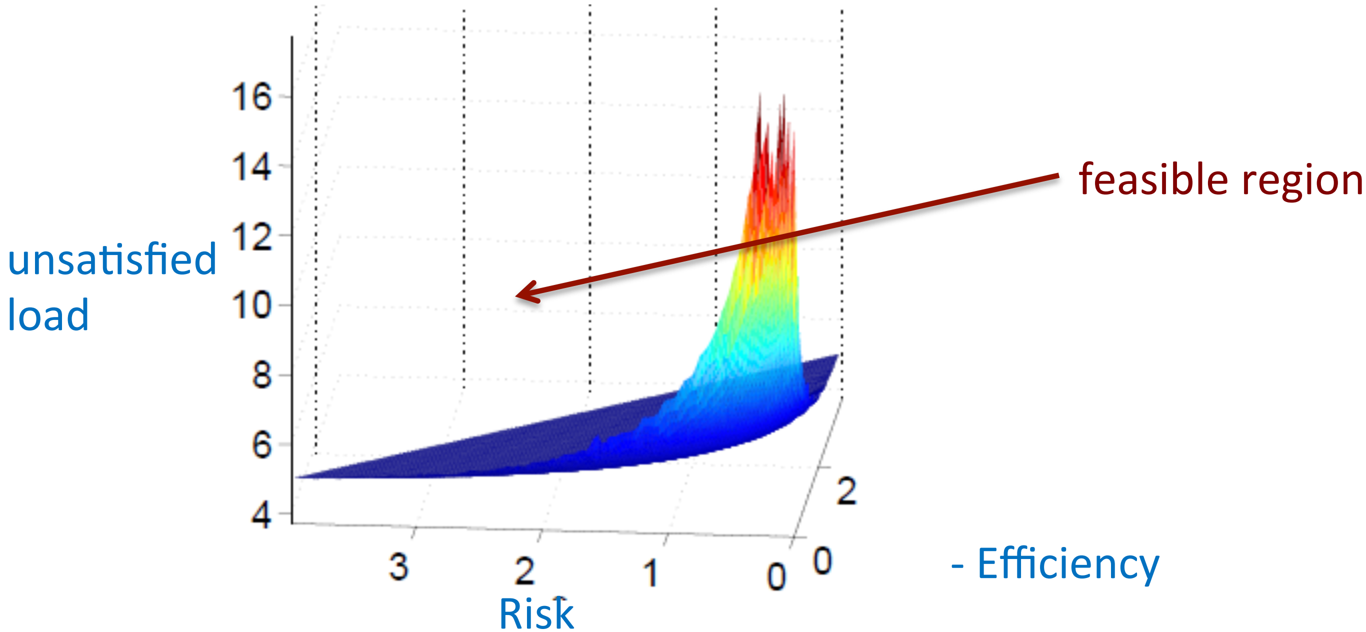


(b)

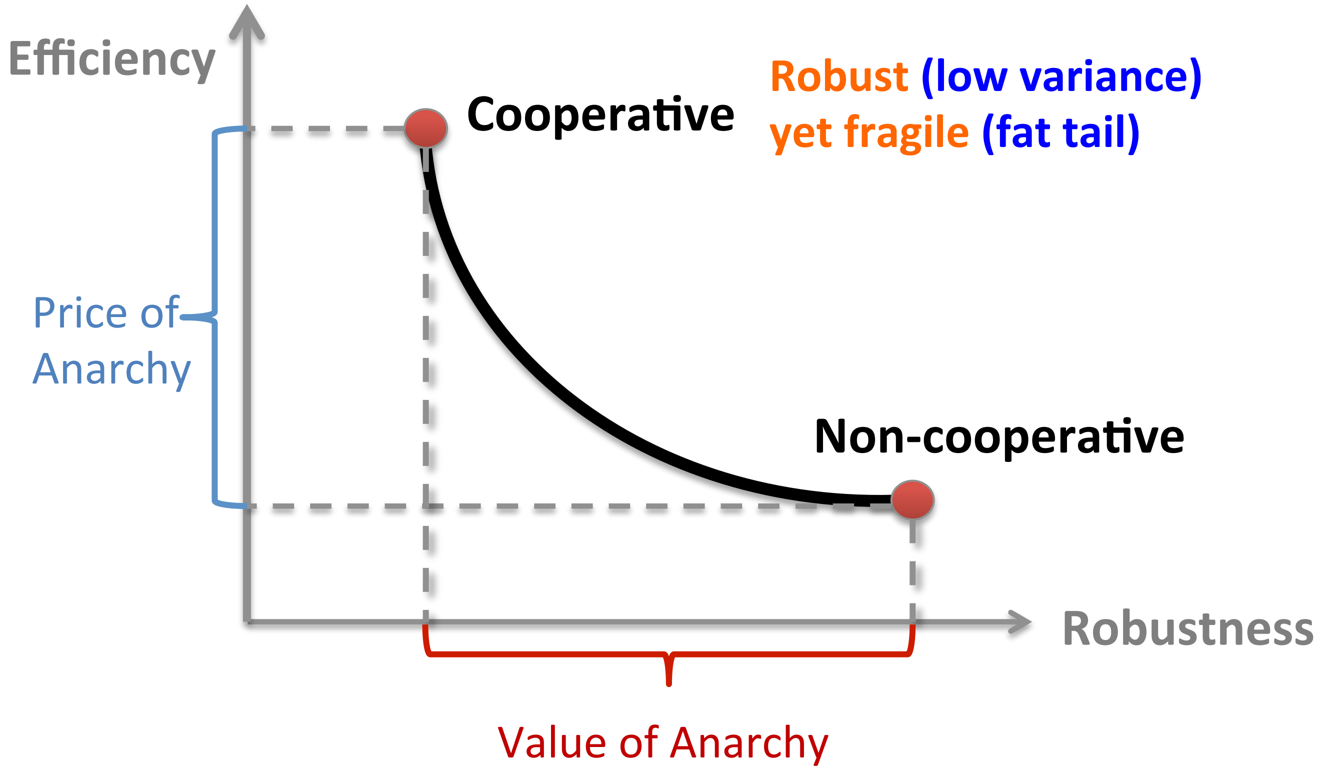
Efficiency frontier characterization
(optimal **F** design with weighted outputs)

Three way tradeoff:

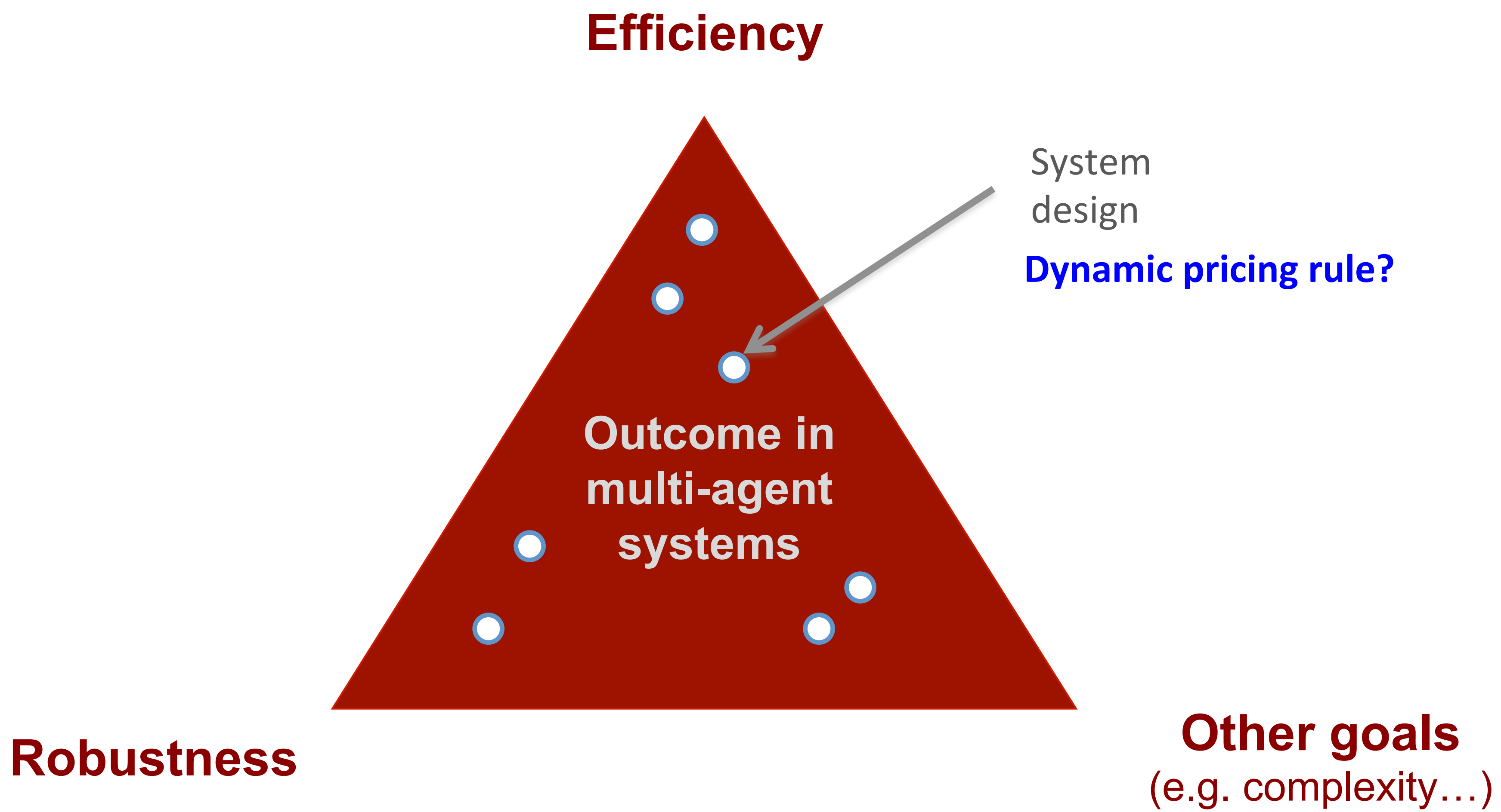
- - Efficiency — aggregate demand 2nd moment
- Risk — aggregate backlog 2nd moment
- Unsatisfied load — aggregate unsatisfied load 2nd moment



Contribution



Conclusion



Efficiency
System
design
Dynamic pricing rule?
Outcome in
multi-agent
systems
Robustness
Other goals
(e.g. complexity…)

Thank you

Case study L=2

System Dynamics

System state: $s(t) = (x(t), d_2(t))$

- Aggregate unshiftable loads $x(t)$
- Consumer arrival with flexible load $d_2(t)$

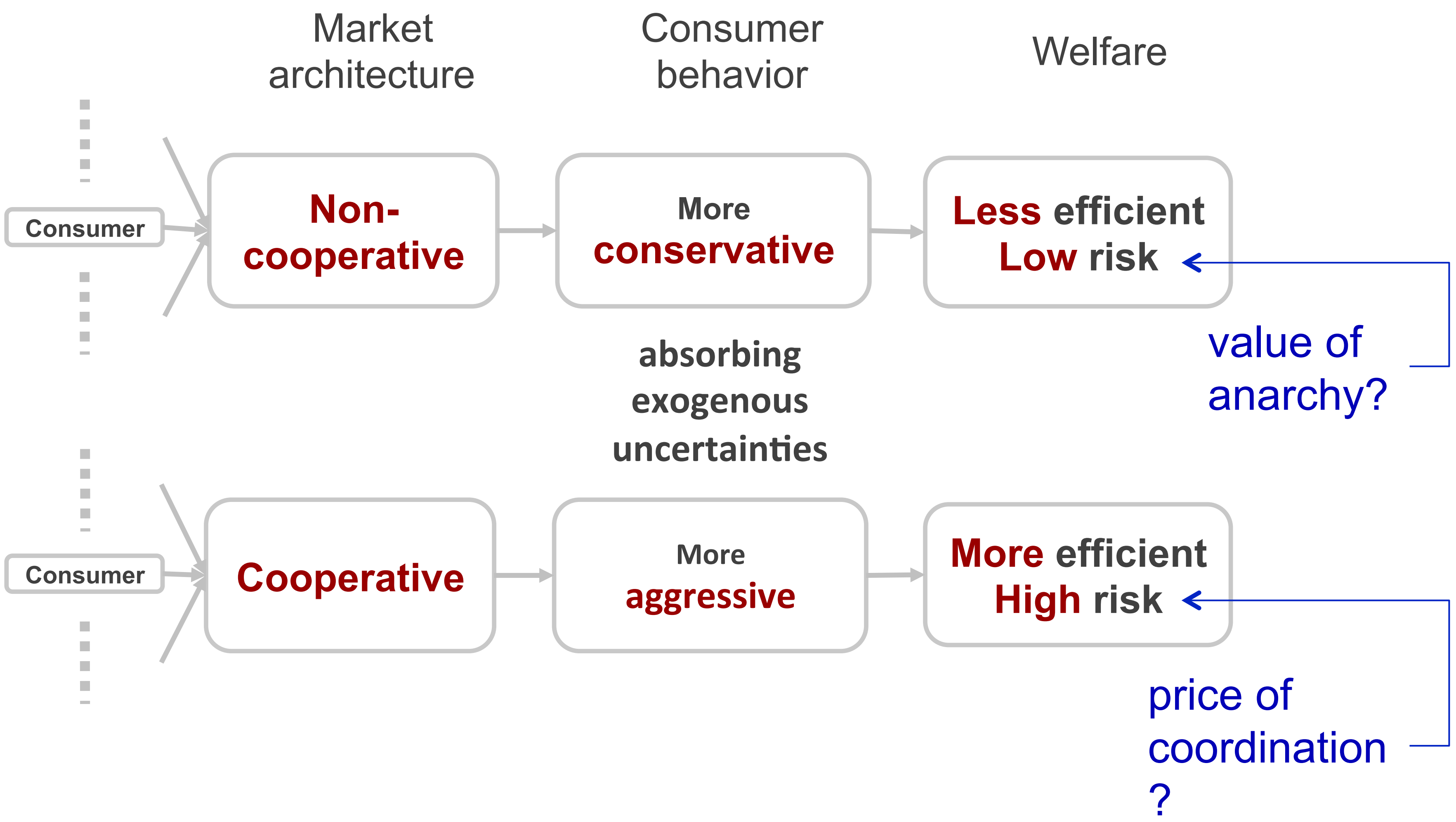
$$\underbrace{x(t)}_{\text{aggregate unshiftable}} = \underbrace{d_1(t)}_{\text{unshiftable arrival at current period}} + \underbrace{d_2(t-1) - u(t-1)}_{\text{leftover from last period's shiftable}}$$

Load scheduling decision:

- At most 1 decision maker at t : the new type 2 agent
- Split load into two periods $(t, t+1)$ based on state information

$$\Big(u(t), \;\; d_2(t) - u(t)\Big)$$

Efficiency and Risk Implications of Architecture



Case study L=2

Welfare measures

market architectures → agent behaviors → aggregate demand process

Efficiency

Producer surplus + Consumer surplus **Variance**

$$W = \underbrace{\mathbf{E}[p(t)U(t) - \frac{1}{2}U(t)^2]}_{W_p} + \underbrace{\mathbf{E}[-p(t)U(t)]}_{W_c} = -\frac{1}{2}\mathbf{E}[U(t)^2]$$

Risk

Probability of aggregate unshiftable load peak **Tail probability**

$$\Pr(x(t) \geq M)$$

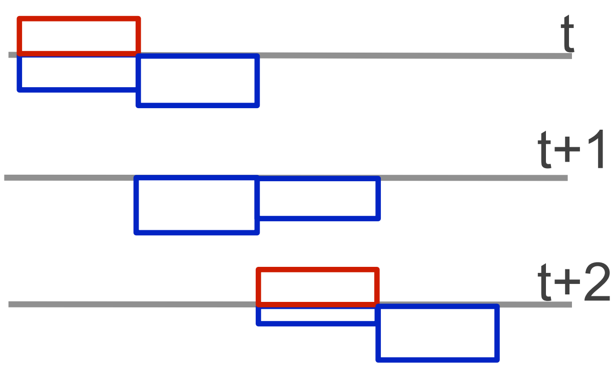
Case study L=2

Solution: Non-Cooperative Case

- Full state (and everything else) information
- No coordination among strategic agents
- Focus on steady state in symmetric equilibria

Symmetric Markov Perfect equilibrium in dynamic stochastic game

$$u^s(x(t), d_2(t)) = \arg\min_u \{p(t)u + \mathbf{E}_t[p(t+1)(d_2(t) - u)]\}$$



$$p(t) = x(t) + u$$
$$p(t+1) = x(t+1) + u^s(x(t+1), d_2(t+1))$$

Overlapping type 2 consumers

Flavor of Stackelberg competition

Case study L=2

Solution: Non-Cooperative Case

Symmetric Markov Perfect equilibrium in dynamic stochastic game

$$u^s(x(t), d_2(t)) = \arg\min_u \{p(t)u + \mathbf{E}_t[p(t+1)(d_2(t) - u)]\}$$

Equilibrium strategy

There exists a unique symmetric MPE with linear equilibrium strategy:

$$u^s(x, d_2) = -a^s x + b^s d_2 + e^s$$

where a^s, b^s, e^s are constants determined by q_1, q_2, μ_1, μ_2 .

Case study L=2

Solution: Cooperative Case

Bellman equation for infinite horizon average cost MDP

$$\lambda^c + V^c(x) = (1-q_2)(x^2 + \mathbf{E}[V^c(d_1)]) + q_2\mathbf{E}[\min_u\{(x+u)^2 + V^c(d_2-u+d_1)\}]$$

Optimal stationary policy

There exists an optimal stationary load scheduling policy:

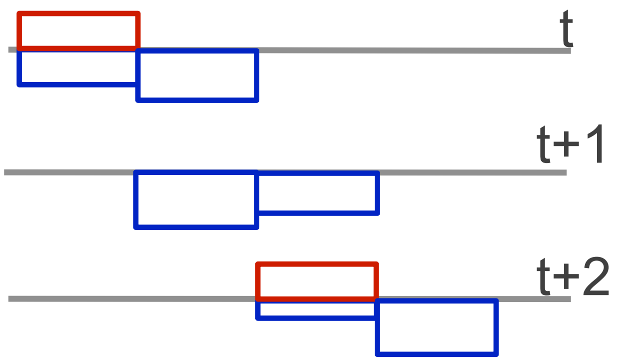
$$u^c(x,d_2) = -a^c x + b^c d_2 + e^c$$

where a^c, b^c, e^c are constants determined by q_1, q_2, μ_1, μ_2.

Case study L=2

Solutions

market architectures → agent behaviors → aggregate demand process



System performance

Efficiency: expected average cost (variance)

Risk: aggregate demand spikes (tail probability)

Non-cooperative	**Cooperative**

A nc > a c, b nc < bc

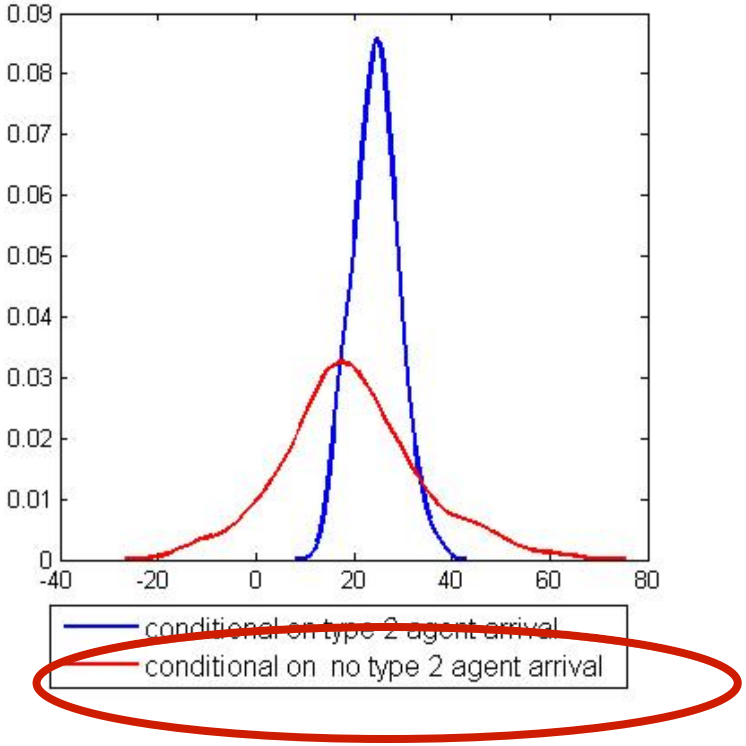
Non-cooperative	Cooperative
Low efficiency	High efficiency
Low risk	High risk

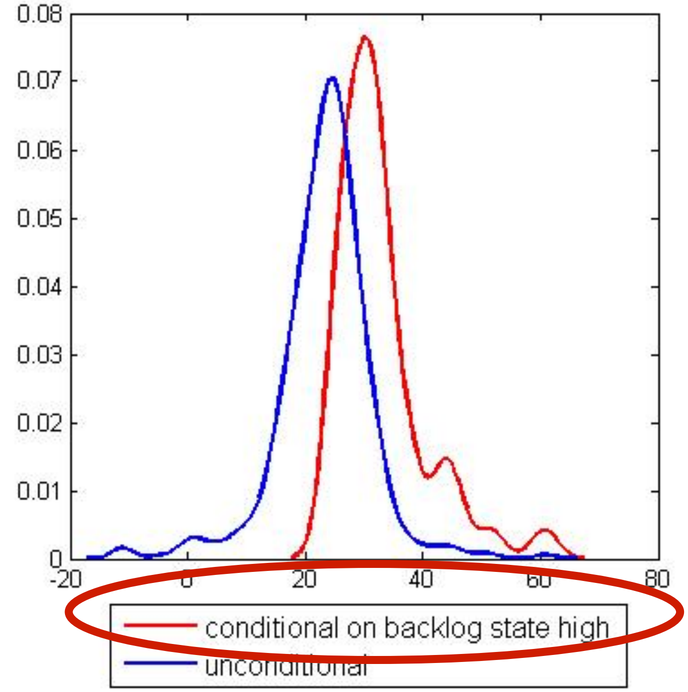
General L analysis

Modified system dynamics

When do spikes occur?

conditional distribution of the aggregate demand





High backlog state

Absence of flexible load