Efficiency and Risk Trade-offs in Dynamic Oligopoly Markets

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Motivation

- How does **endogenous risk** emerge in multi-agent dynamic system?
- What’s the impact of local incentives on system-wide efficiency and risk?

Exogenous shocks VS endogenous risk
Tail risk, aggregate demand / price spikes

market architectures $\rightarrow$ agent behaviors $\rightarrow$ aggregate outcome

Efficiency VS Risk

Tradeoff
Contribution

- **Efficiency**
  - Cooperative: Reduce externality, more efficient
  - Non-cooperative: More vulnerable to some exogenous shocks

- **Robustness**
  - Cooperative: More system dynamics, uncertainties, frictions (deadline constraints)
  - Non-cooperative: Less system dynamics, uncertainties, frictions (deadline constraints)

- **Price of Anarchy**
  - Cooperative: Lower price of anarchy
  - Non-cooperative: Higher price of anarchy
• Previous works on **endogenous risk**
  • Heterogeneous beliefs [1]
  • Failure of the agents to rationalize feedback links [2]

• Our work
  • **Agents are fully aware of pricing mechanism,**
    **have perfect information of system state,**
    **form rational expectations about other agents in the market.**

  • **Market architecture → Endogenous risk**
  • **Tradeoff**

• **Approach**
  • **Case study : Dynamic stochastic game, MDP**
  • **LTI reformulation and tradeoff analysis**

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[1] The leverage cycle (Geanakoplos 2009)
Setup

Agent arrival
- L types (deadline constraint)
- Uncertainty
  - Bernoulli arrival
  - Workload distribution $d_l(t), l \in \{1, \ldots, L\}$

Decision
- State information
- Load scheduling

Cost
$$\mathbb{E}\left[\sum_{\tau=t}^{t+l} u_i(t)p(t)\right]$$

Pricing
$$p(t) = \sum_i u_i(t)$$

System performance
- Efficiency: expected average cost (variance of aggregate demand process)
- Risk: aggregate demand spikes (tail probability of aggregate demand process)
Setup

Exogenous demand shocks

$u(t)$

Market
state: backlogged loads of existing agents

Dynamic demand scheduling

Supply Schedule
Price (marginal cost)

Market architectural properties
Cooperative / non-cooperative
Pricing method
Risk sensitivity
Information
**Setup**

**Applications:**
- Consumer response to real-time pricing in power grids
- Load scheduling in cloud computing market
- Multi-portfolio execution problem
- Consumption risk sharing

**Market**
- State: backlogged loads of existing agents

**Dynamic demand scheduling**

**Exogenous demand shocks**

**Supply Schedule**
- Price (marginal cost)

\[ u(t) \]
Case study $L=2$

Solutions

**market architectures → agent behaviors → aggregate demand process**

- At most 1 decision maker at each period
  \[
  \begin{pmatrix}
  u(t), & d_2(t) - u(t)
  \end{pmatrix}
  \]
- Linear quadratic
  \[
  u^s(x, d_2) = -a^s x + b^s d_2 + e^s
  \]

**Non-cooperative**

- Dynamic stochastic game
- Markov perfect equilibrium

**Cooperative**

- Infinite horizon average cost MDP

\[
\begin{align*}
a^{nc} & < a^c \\
b^{nc} & > b^c
\end{align*}
\]

In cooperative scheme, agents respond more aggressively to other agents’ shock

- Low efficiency
  - Low risk
- High efficiency
  - High risk
Case study L=2

Results

Aggregate demand sample path

small time scale

large time scale

Cooperative
Non-cooperative
Case study \( L=2 \)

Results

Aggregate demand distribution

- **Linear scale**
- **Log scale**

**Efficiency**

**Risk**

Cooperative

Non-cooperative
General L analysis
Modified system dynamics

- Jump linear system
- No closed form solution

\[
\begin{align*}
  x(t+1) &= R_1(x(t) - u(t)) + R_2 d(t) \\
  o(t+1) &= R_1 o(t) + R_2 h(t)
\end{align*}
\]

Risk of spikes
- High backlog of load
- Absence of flexible load (Bernoulli)

Cooperative load scheduling
- less externality
- more rely on each other
- more heavily use of backlog
General L analysis
Modified system dynamics

- Focus on linear dynamics
- Relax deadline constraints
- Substitute risk measure

- LTI system
- Multi-objective optimization
  - Efficiency aggregate demand 2nd moment
  - Risk aggregate backlog 2nd moment
  - Aggregate unsatisfied load 2nd moment

\[
\begin{align*}
x(0) &= h(t) \\
d(t) &= u(t) \\
x(t+1) &= R_1(x(t) - u(t)) + R_2d(t) \\
o(t+1) &= R_1o(t) + R_2h(t)
\end{align*}
\]

\[
U(t) \quad d(t) \quad x(t), o(t) \quad u(t)
\]

\[
\begin{bmatrix}
R_1 \\
0 \\
\alpha_2e' \\
\alpha_3e'_L
\end{bmatrix}
\begin{bmatrix}
0 \\
\alpha_1e' \\
0 \\
\alpha_3e'_L
\end{bmatrix}
\begin{bmatrix}
R_2 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
-x_1 \\
0 \\
-x_2 \\
-x_3
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\]

\[
l(t) = Fx(t)
\]
General L analysis
Results

**Efficiency frontier characterization**
(optimal F design with weighted outputs)

Three way tradeoff:
- Efficiency
- Risk
- Unsatisfied load

aggregate demand 2\textsuperscript{nd} moment
aggregate backlog 2\textsuperscript{nd} moment
aggregate unsatisfied load 2\textsuperscript{nd} moment

feasible region
unsatisfied load

- Efficiency

Risk

- Efficiency
**Contribution**

- **Efficiency**
- **Robustness**

<table>
<thead>
<tr>
<th>Price of Anarchy</th>
<th>Value of Anarchy</th>
</tr>
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<tbody>
<tr>
<td>Cooperative</td>
<td>Robust (low variance) yet fragile (fat tail)</td>
</tr>
<tr>
<td>Non-cooperative</td>
<td></td>
</tr>
</tbody>
</table>

- **Efficiency** axis
- **Robustness** axis
- **Value of Anarchy** axis
Conclusion

Efficiency

Robustness

Outcome in multi-agent systems

Other goals
(e.g. complexity…)

System design
Dynamic pricing rule?
Thank you
Case study L=2
System Dynamics

**System state:** $s(t) = (x(t), d_2(t))$

- Aggregate unshiftable loads $x(t)$
- Consumer arrival with flexible load $d_2(t)$

\[
\begin{align*}
  x(t) &= \underbrace{d_1(t)}_{\text{unshiftable arrival at current period}} + \underbrace{d_2(t - 1) - u(t - 1)}_{\text{leftover from last period’s shiftable load}} \\
  &\quad + \underbrace{\text{aggregate unshiftable}}_{\text{load}}
\end{align*}
\]

**Load scheduling decision:**

- At most 1 decision maker at $t$ : the new type 2 agent
- Split load into two periods $(t, t+1)$ based on state information

\[
\begin{pmatrix}
  u(t), \\ d_2(t) - u(t)
\end{pmatrix}
\]
Efficiency and Risk Implications of Architecture

Market architecture

Consumer behavior

Welfare

**Non-cooperative**
- More conservative
- Less efficient
- Low risk

**Cooperative**
- More aggressive
- More efficient
- High risk

Consumer

absorbing exogenous uncertainties

value of anarchy?

price of coordination?
Case study L=2
Welfare measures

market architectures $\rightarrow$ agent behaviors $\rightarrow$ aggregate demand process

Efficiency

Producer surplus $+$ Consumer surplus  Variance

$$W = \mathbb{E}[p(t)U(t) - \frac{1}{2}U(t)^2] + \mathbb{E}[-p(t)U(t)] = -\frac{1}{2}\mathbb{E}[U(t)^2]$$

Risk

Probability of aggregate unshiftable load peak  Tail probability

$$\Pr(x(t) \geq M)$$
Case study L=2
Solution: Non-Cooperative Case

- Full state (and everything else) information
- No coordination among strategic agents
- Focus on steady state in symmetric equilibria

Symmetric Markov Perfect equilibrium in dynamic stochastic game

\[ u^s(x(t), d_2(t)) = \arg\min_u \{ p(t)u + E_t[p(t+1)(d_2(t) - u)] \} \]

\[ p(t) = x(t) + u \]
\[ p(t+1) = x(t+1) + u^s(x(t+1), d_2(t+1)) \]

Overlapping type 2 consumers
Flavor of Stackelberg competition
Symmetric Markov Perfect equilibrium in dynamic stochastic game

\[ u^s(x(t), d_2(t)) = \arg \min_u \{ p(t)u + E_t[p(t + 1)(d_2(t) - u)] \} \]

Equilibrium strategy

There exists a unique symmetric MPE with linear equilibrium strategy:

\[ u^s(x, d_2) = -a^s x + b^s d_2 + e^s \]

where \( a^s, b^s, e^s \) are constants determined by \( q_1, q_2, \mu_1, \mu_2 \).
Bellman equation for infinite horizon average cost MDP

\[ \lambda^c + V^c(x) = (1 - q_2)(x^2 + \mathbb{E}[V^c(d_1)]) + q_2 \mathbb{E}[\min_u \{(x + u)^2 + V^c(d_2 - u + d_1)\}] \]

Optimal stationary policy

There exists an optimal stationary load scheduling policy:

\[ u^c(x, d_2) = -\alpha^c x + b^c d_2 + e^c \]

where \( \alpha^c, b^c, e^c \) are constants determined by \( q_1, q_2, \mu_1, \mu_2 \).
Case study L=2

Solutions

market architectures → agent behaviors → aggregate demand process

System performance

**Efficiency:** expected average cost (variance)

**Risk:** aggregate demand spikes (tail probability)

Non-cooperative

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Cooperative

<table>
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General L analysis
Modified system dynamics

When do spikes occur?
conditional distribution of the aggregate demand

Absence of flexible load

High backlog state