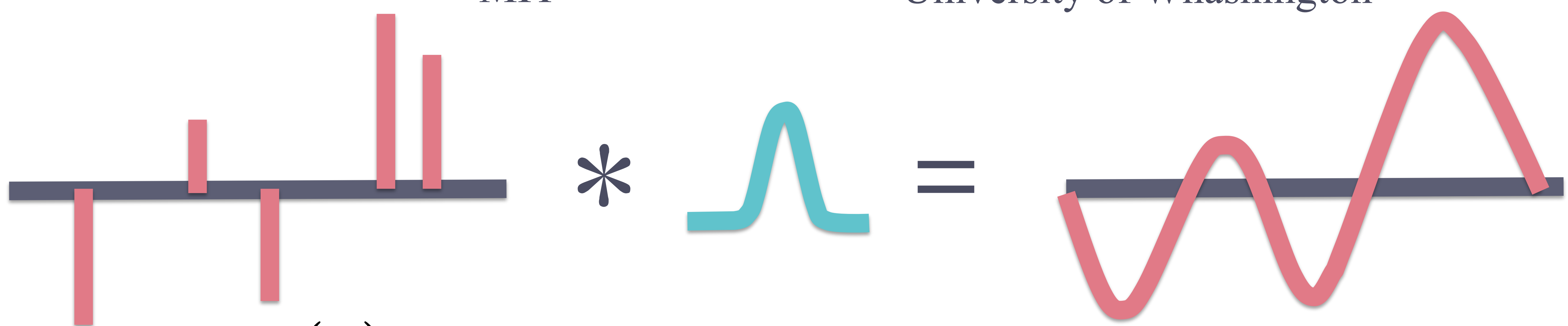
Super-resolution off the grid

Qingqing Huang *
MIT

Sham M Kakade
University of Whashington



$x(t)$

band limited observation

✧ **Problem:** recover a superposition of k point sources in d-dim

$$x(t) = \sum_{j=1}^{k} w_j \delta_{\mu^{(j)}}.$$

using bandlimited and noise corrupted Fourier measurements.

(Fourier measurements) $f(s) = \int_{t \in \mathbb{R}^d} e^{i\pi <t,s>} x(\mathrm{d}t) = \sum_{j=1}^{k} w_j e^{i\pi <\mu^{(j)},s>}.$

(Measurement noise) $\widetilde{f}(s) = f(s) + z(s), \quad |z(s)| \leq \epsilon_z, \forall s.$

Super-resolution off the grid

$x(t) = \sum_{j=1}^{k} w_j \delta_{\mu^{(j)}}.$ $\qquad \widetilde{f}(s) = \sum_{j=1}^{k} w_j e^{i\pi<\mu^{(j)},s>} + z(s), \quad \forall s$

- **Problem**
  - Take measurements at different s , try to recover $\mu^{(j)}$'s
- **Goal:**
  - coarse measurements (cutoff frequency $||s|| < R$)
  - use a small number of Fourier measurements: m
  - algorithm runs quickly

Super-resolution off the grid

✧ Minimum separation

$$\Delta = \min_{j\neq j'} \|\mu^{(j)} - \mu^{(j')}\|_2$$

$$\Delta_\infty = \min_{j\neq j'} \|\mu^{(j)} - \mu^{(j')}\|_\infty$$

	$\|s\| < R$	m
✧ **Prony's method** (Matrix−Pencil / MUSIC / ESPRIT)	no stability guarantee	
✧ **Super-resolution** (SDP)	$\frac{1}{\Delta_\infty}$	$poly((\frac{1}{\Delta_\infty})^d, k)$
✧ **Our algorithm**	$\frac{1}{\Delta}$	$(k+d)^2$

✧ Algorithmic idea: Prony's method on **Random** samples