**Problem**: recover a superposition of $k$ point sources in $d$-dim using bandlimited and noise corrupted Fourier measurements.

(Fourier measurements) \[ f(s) = \int_{t \in \mathbb{R}^d} e^{i \pi <t,s>} x(dt) = \sum_{j=1}^{k} w_j e^{i \pi <\mu(j),s>} . \]

(Measurement noise) \[ \tilde{f}(s) = f(s) + z(s), \quad |z(s)| \leq \epsilon_z , \forall s. \]
Super-resolution off the grid

\[ x(t) = \sum_{j=1}^{k} w_j \delta_{\mu^{(j)}}. \quad \tilde{f}(s) = \sum_{j=1}^{k} w_j e^{i\pi \langle \mu^{(j)}, s \rangle} + z(s), \quad \forall s \]

✧ Problem

✧ Take measurements at different \( s \), try to recover \( \mu^{(j)}, s \)

✧ Goal:

✧ coarse measurements (cutoff frequency \( ||s|| < R \))
✧ use a small number of Fourier measurements: \( m \)
✧ algorithm runs quickly
**Super-resolution off the grid**

- Minimum separation
  \[ \Delta = \min_{j \neq j'} \| \mu(j) - \mu(j') \|_2 \]
  \[ \Delta_\infty = \min_{j \neq j'} \| \mu(j) - \mu(j') \|_\infty \]

|                | \( ||s|| < R \) | \( m \) |
|----------------|-----------------|--------|
| **Prony’s method**  | no stability guarantee |        |
| (Matrix-Pencil / MUSIC / ESPRIT) |     |        |
| **Super-resolution (SDP)**          | \( \frac{1}{\Delta_\infty} \) | \( \text{poly}( \left( \frac{1}{\Delta_\infty} \right)^d, k) \) |
| **Our algorithm**                  | \( \frac{1}{\Delta} \) | \( (k + d)^2 \) |

- Algorithmic idea: Prony’s method on **Random** samples

NIPS 2015 Spotlight